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Final Exam MTH 418, Spring 2016

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QUESTION 1. (i) We know that if H is a tree with n vertices, then H must have n -1 edges. Use the fact that every tree is a planar to prove that every tree of order n must have exactly n -1 edges (one line to two lines proof!)

(ii) Let H be a connected graph with 5 vertices and with nondecreasing associated sequence of degrees $4 \geq 2 \geq 2 \geq 2 \geq 2$.

a. What is the size of H?

b. Show that H is a planar.

c. Find $\chi(H)$

d. Find $\chi'(H)$

e. Find $\kappa(H)$

f. Find $\kappa'(H)$.

g. It is clear that H is Eulerian, is it Hamiltonian?

h. Draw the $cl(H)$, i. e., the closure of H.

(iii) Find $\gamma(C_8)$ and $\gamma(\overline{C_8})$ (domination number for C_8 and domination number for the complement of C_8)

(iv) Convince me that $\overline{P_n}$ is connected for every $n \geq 4$.

(v) Let T be a tree of order 6. If $T = K_{1,5}$, then show that $\overline{T}$ is not planar. If $T \neq K_{1,5}$, then show that $\overline{T}$ is a planar. Hence we conclude that $\overline{P_6}$ is a planar.

(vi) Convince me that $\overline{P_7}$ is not a planar (Hence we conclude that $\overline{P_n}$ is not a planar for every $n \geq 7$).

(vii) Let H be a connected graph such that $\chi(H) = \chi'(H) = \triangle + 1$. Find all possibilities of H.

(viii) Let F be a $B_{3,3}$ such that four vertices, each is of degree 3, and exactly two vertices, each is of degree 2. By drawing F, convince me the F is a kissing graph of circles.

(ix) Let $H = K_5$ and $M = K_3$. Consider the product graph $L = H \times M$. Find the degree of each vertex. Find $\chi'(L)$. Convince me that L is not a kissing graph of circles.

(x) Consider that graph Q_4, (cubic-graph with 2^4 vertices). Convince me that Q_4 has an induced subgraph of order 5 that is a tree. Construct such induced subgraph.

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(i) Since a tree has no cycles; then; $F_{all}(T)=1$ (The Unbounded face)

Hence; Since it is planar; $|V(T)| - |E(T)| + F(all) = 2$

$\iff n - |E(T)| = 1 \iff \boxed{|E(T)| = n-1}$

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Excellent

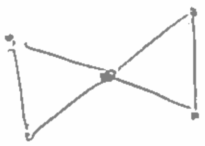
(ii) Connected graph; $4 \geq 2 \geq 2 \geq 2 \geq 2$.

ⓐ Size of H; $\sum_{v\in V(H)} \deg(v) = 2|E(H)|$

$\Rightarrow 4+2+2+2+2 = 2|E(H)|$

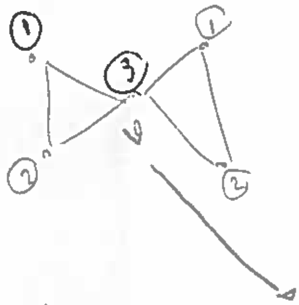
$\boxed{|E(H)| = 6}$

ⓑ We can draw H this way



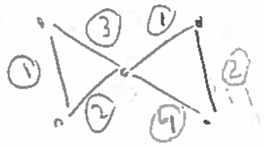
Hence it is planar;

ⓒ



$\chi(H) = 3$

ⓓ



$\chi'(H) = 4$

ⓔ $K(H) = 1$; remove v from above;

ⓕ $K'(H) = 2$, remove two edges incident to v.

ⓖ No; H is not hamiltonian;

ⓕ There are vertices $u, v \in V(H)$ such that $\deg(u) + \deg(v) \geq 5$ and $u - v \notin V(H)$;

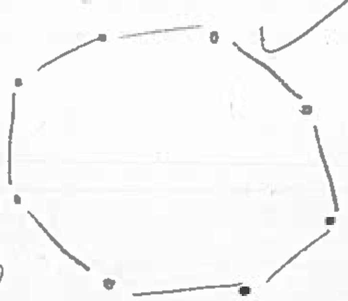
but we have G_1 as

which is hamiltonian

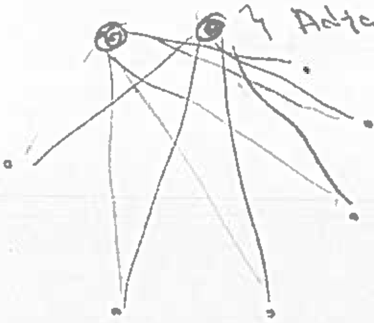
but hypothesis is not fully satisfied

(iii) $\gamma(C_8) = \left\lceil \frac{8}{3} \right\rceil = 3$

$\gamma(\overline{C_8}) = 2$.



$\Rightarrow$ $\overline{C_8}$



Adjacent to all other vertices

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(iv) $\overline{P}_n$ is connected.

Suppose $\overline{P}_n$ is disconnected

then $\overline{(\overline{P}_n)}$ is connected and $diam(\overline{P}_n) \le 2$

but $\overline{\overline{P}}_n = P_n$ and $diam(P_n) \geq 2$

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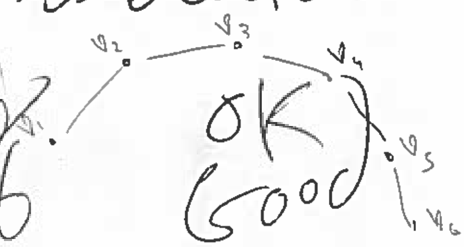
Since it is an open walk;

$\Rightarrow \overline{P}_n$ is connected for $n \geq 4$.

(v) ~~It is a tree of order 6~~



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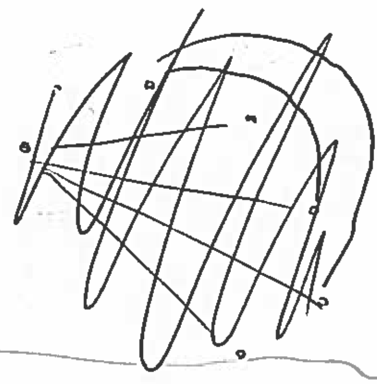
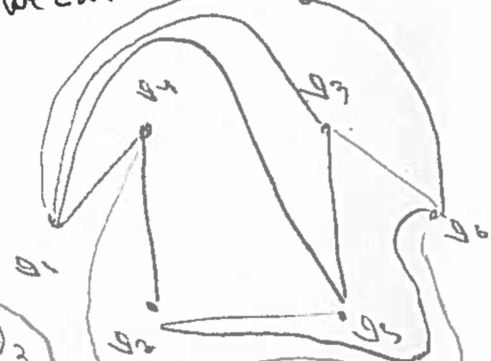
OK Good

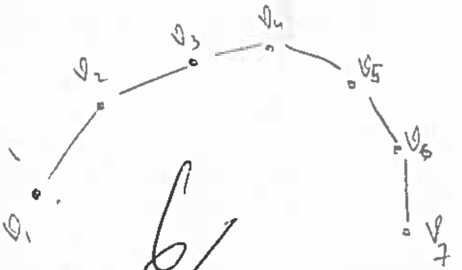


$\therefore$ Planar.

we can draw $\overline{P}_6$ as.



(vi) $\overline{P}_7$ is not planar.



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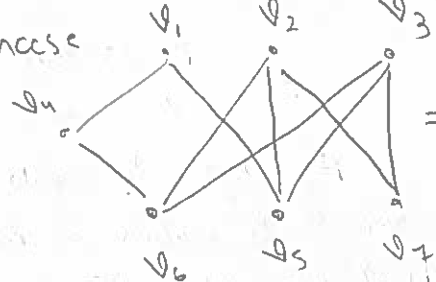
Choose



$\Rightarrow$ Subgraph of $\overline{P}_7$

$\Rightarrow$ Subdivision of $K_{3,3}$

$\Rightarrow$ Not Planar

(vii) $X(H) = X'(H) = \Delta(H)+1$

We know $X(H) = \Delta(H)+1$ iff $H=C_n$ (n is odd) or $H=K_n$

Here; We know that

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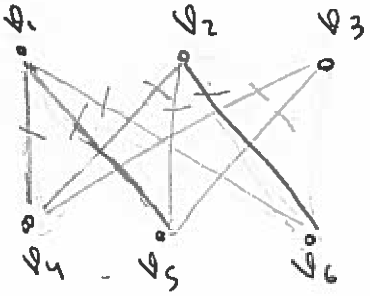
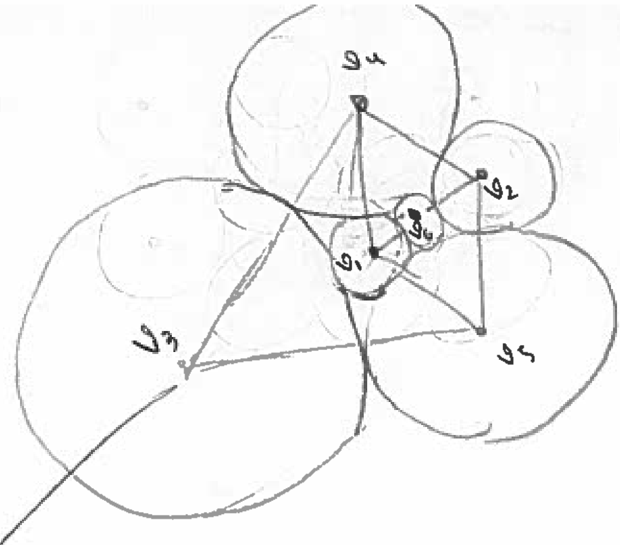
$X'(C_n) = 3 = \Delta(C_n)+1$ when n is odd

and $X'(K_n) = n$ when n is odd $\Rightarrow$ ✓ H could be C_n when n is odd

~~or K_n when n is odd~~

(viii) $K_{3,3}$





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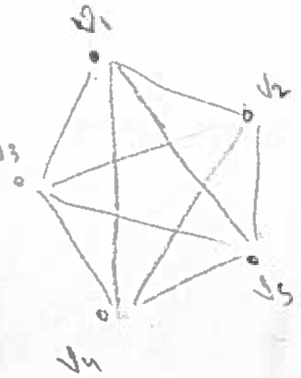
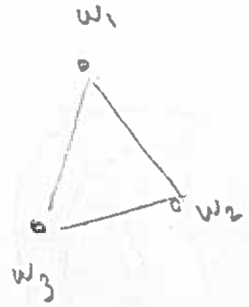
(ix) $H = K_5$ and $M = K_3$





For $H \times M$;
the degree of any vertex is 6
and we have $(3)(5)$ vertices

Hence; $\Delta(L) = 6$
and $\left\lfloor \frac{15}{2} \right\rfloor = 7$

and; we have;

$$|E(L)| = \frac{6(15)}{2} = 45; \text{ hence;}$$

$$|E(L)| > \Delta(L) \left\lfloor \frac{|V(L)|}{2} \right\rfloor$$

$$45 > 42$$

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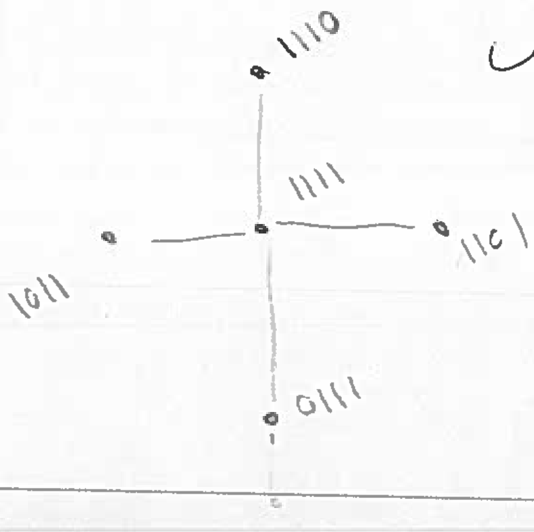
$\Rightarrow \chi'(L) = \Delta(L) + 1 = 7.$

Clearly; K_5 is a Subgraph of L; $\Rightarrow$ L is not planar

$\Rightarrow$ Not a kissing graph of circles.

(x) Q_4; Take the Subgraph as.



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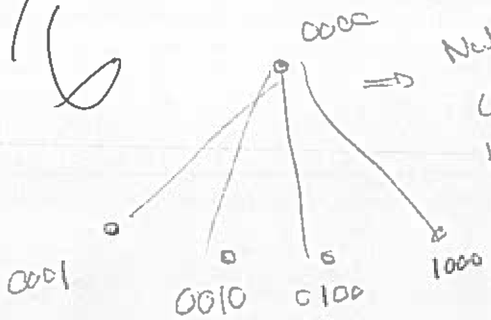
OR//



$\Rightarrow$ Note that; Simply choose a vertex, v in Q_4 then choose all the vertices adjacent to it, if two are adjacent! then choose a different vertex v.